59. Proposed by **WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

Show that the tangent plane at any point of the surface $a^2x^2+b^2y^2+c^2z^2=2bcyz+2acxz+2abxy$ intersects the surface $ayz+bzx+cxy=0$ in two straight lines at right angles to one another.

CALCULUS.

Conducted by **J. M. COLAW**, Monterey, Va. All contributions to this department should be sent to him.

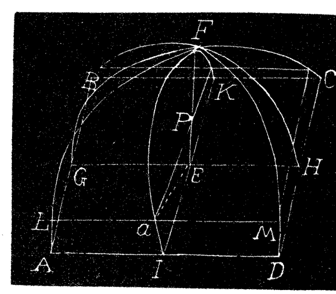
SOLUTIONS OF PROBLEMS.

47. Proposed by **Prof. J. SCHEFFER, A. M., Hagerstown, Maryland.**

The floor of a vault forms a square, and all sections parallel to it are squares. The two vertical sections through the middle points of the opposite sides of the floor are equal semi-circles. Find the convex surface and the volume of the vault.

I. Solution by **C. W. M. BLACK, Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts; O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland; A. H. HOLMES, Brunswick, Maine, and the PROPOSER.**

Let $ABCD$ represent the base square, side$=2a$, and KEI and GFH the two equal semi-circles, radius$=a$. Let $LMNO$ be another square parallel to the base square, and at the distance $PE=x$ from it. The area of $LMNO$ is$=4(a^2-x^2)$,



$$\therefore \text{ Vol.}=4\int_0^a (a^2-x^2)dx=\tfrac{8}{3}a^3.$$

Denoting $\angle PEQ$ by θ, we have for the surface

$$\int_0^{\frac{1}{2}\pi} 8a\cos\theta d(a\theta)=8a^2.$$ Or for the volume, $dV=4a^2\cos^2\theta dx$, where x is the vertical distance. $x=a\sin\theta$; $dx=a\cos\theta d\theta$.

$$\therefore\ V=4a^3\int_0^{\frac{1}{2}\pi}\cos^3\theta d\theta,\ =a^3\int_0^{\frac{1}{2}\pi}(\cos3\theta+3\cos\theta)d\theta=\frac{8a^3}{3}.$$

II. Solution by **BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, Ohio.**

The convex surface of the vault is equivalent to the surface of a right cir-

cular cylinder intercepted by another right circular cylinder, their axes intersecting at right angles, the two cylinders being equal, and the diameter of each equal to that of the vertical sections of the vault.

$\therefore$ Letting the radius$=a$, $S=8a\int_0^a\int_0^{\sqrt{a^2-x^2}}\frac{dxdy}{\sqrt{a^2-x^2}}=8a^2$, the equations of the cylinders being $x^2+z^2=a^2$, and $x^2+y^2=a^2$.

The volume is equivalent to that of four wedges cut from the cylinder, $x^2+y^2=a^2$, by the planes, $z=0$, and $z=x$.

$$\therefore\ V=8\int_0^a\int_0^{\sqrt{a^2-x^2}}\int_0^x dxdydz=\frac{8a^3}{3}.$$

Also solved by *E. L. SHERWOOD* and *G. B. M. ZERR*.

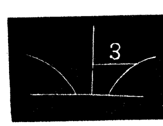
48. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.**

I have a circular section basin 12 inches in perpendicular height; the diameters are as follows: At base, 2 inches; one inch perpendicular height, 6 inches; two inches perpendicular height, 18 inches; three inches perpendicular height, 54 inches; and so on, the diameter being trebled for every inch in height. After a rain the water in the basin is six inches deep, what was the rainfall?

I. **Solution by E. L. SHERWOOD, A. M., Professor of Mathematics, Mississippi Normal College, Houston, Mississippi; C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts, and the PROPOSER.**

The basin is generated by revolving the curve $x=3^y$ about the axis of y.

$$\therefore\ \text{Volume of water}=\pi\int_0^6 x^2dy=\pi\int_0^6 3^{2y}dy.$$



$$\therefore\ V=\pi\frac{3^{12}-1}{2\log3}=\frac{531440\pi}{2\log3}.$$

Let x=depth of rain-fall, then since radius of top of basin$=3^{12}$, $V=\pi3^{24}x$.

$$\therefore\ x=\frac{565720}{282429536481\log3}=.00000086\text{ inches}.$$

II. **Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.**

Call x the length of any radius, and y the vertical distance, y being 1 at the bottom of the basin. Then the equation of side of basin is $x=3^{y-1}$,

$$dV=\pi x^2dy,\ V=\pi\int_1^y 3^{2y-2}dy=\frac{\pi[3^{12}-1]}{3\log_e3}.$$

The radius of upper base$=3^{12}$. Call R the rainfall, then